SANTOSH Academia
IIT-JEE | NEET | Foundation

Answers & Solutions

Time : 3 hrs. M.M. : 300

for

JEE (Main)-2025 Phase-1

[Computer Based Test (CBT) mode]

(Mathematics, Physics and Chemistry)

28/01/2025

Morning

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (MPC) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three** Parts. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions:This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0$, $a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n$, $n = 0, 1, 2, 3, \ldots$. Then $\sum_{k=1}^{100} a_k$ is equal to

(1) $3a_{100} - 100$ (2) $3a_{100} + 100$

(3) $3a_{99} - 100$ (4) $3a_{99} + 100$

Answer (1)

Sol. $2a_{n+2} = 5a_{n+1} - 3a_n$, $n = 0, 1, 2, \cdots$ using characteristic equation

$2x^2 = 5x - 3$

$\Rightarrow \quad x = 1, \frac{3}{2}$

$\Rightarrow \quad a_n = p(1)^n + q\left(\frac{3}{2}\right)^n$

$\Rightarrow \quad a_0 = p + q = 0$

$a_1 = p + \frac{3}{2}q = \frac{1}{2} \Rightarrow q = 1,\ p = -1$

$\Rightarrow \quad a_n = \left(\frac{3}{2}\right)^n - 1$

$$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100}\left(\frac{3}{2}\right)^k - \sum_{k=1}^{100}(1)$$

$$= \frac{\left(\frac{3}{2}\right)\left[\left(\frac{3}{2}\right)^{100} - 1\right]}{\left(\frac{3}{2} - 1\right)} - 100$$

$$= 3\left[\left(\frac{3}{2}\right)^{100} - 1\right] - 100$$

$= 3a_{100} - 100$

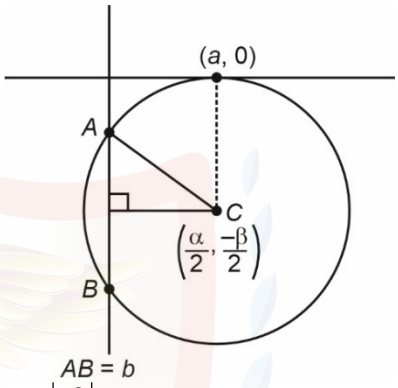
2. Let the equation of the circle, which touches x-axis at the point $(a, 0)$, $a > 0$ and cuts off an intercept of length b on y-axis be $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$. If the circle lies below x-axis, then the ordered pair $(2a, b^2)$ is equal to

(1) $(\alpha, \beta^2 + 4\gamma)$ (2) $(\gamma, \beta^2 - 4\alpha)$

(3) $(\gamma, \beta^2 + 4\alpha)$ (4) $(\alpha, \beta^2 - 4\gamma)$

Answer (4)

Sol.



$AB = b$

$r = \left|\frac{-\beta}{2}\right|$

$a = \frac{\alpha}{2} \Rightarrow 2a = \alpha$

$\Rightarrow \quad r^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{\alpha}{2}\right)^2$

$\Rightarrow \quad \frac{\beta^2}{4} = \frac{b^2}{4} + a^2$

$\Rightarrow \quad b^2 = \beta^2 - 4a^2 = \beta^2 - (2a)^2 = \beta^2 - \alpha^2$

$r^2 = \frac{\beta^2}{4} = \frac{\alpha^2}{4} + \frac{\beta^2}{4} - \gamma \Rightarrow \gamma = \frac{\alpha^2}{4}$

$\Rightarrow \quad b^2 = \beta^2 - 4\gamma$

$(2a, b^2) = (\alpha, \beta^2 - 4\gamma)$

3. Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}$, $(i = \sqrt{-1})$ is non-zero, equals

(1) $\frac{2}{3}$ (2) $\frac{1}{4}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Answer (3)

Sol. Let $k_1 = 4\lambda_1 + r_1$, $r_i \in \{0, 1, 2, 3\}$

$k_2 = 4\lambda_2 + r_2$

$(i)^{k_1} + (i)^{k_2} = (i)^{r_1} + (i)^{r_2}$

$(i)^{r_1} \in \{1, i, -1, -i\}$

Zero $\Rightarrow$ 1, (–1) pair $\Rightarrow$ $\left\{\begin{matrix} 1, & -1 \\ i, & -i \\ -i, & +i \\ -1, & 1 \end{matrix}\right\}$

i, ($-i$) pair

Zero probability = $\dfrac{4}{{}^4C_1 \cdot {}^4C_1} = \dfrac{1}{4}$

Probability (non-zero) $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

4. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8, is

(1) 5719 (2) 4608

(3) 5720 (4) 4607

Answer (4)

Sol. $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}\ \underline{e}$

$a \geq 5$

all b, c, d, e are not 0

(i) $a = 5$

$5\ \_\ \_\ \_\ \underline{e} \Rightarrow e \in \{3, 2, 1, 0\}$

$8^3 \cdot 4 - 1$ ways

(ii) $a = 6$

$6\ \_\ \_\ \_\ \underline{e} \Rightarrow e \in \{2, 1, 0\}$

$8^3 \cdot 3$ ways

(iii) $7\ \_\ \_\ \_\ \underline{e} \Rightarrow e \in \{1, 0\}$

$8^3 \cdot 2$ ways

Total = $8^3[4 + 3 + 2] - 1$

= 4607 numbers

5. Let ${}^nC_{r-1} = 28$, ${}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let A (4cos$t$, 4sin$t$), B (2sin$t$, –2cos$t$) and C (3r–n, $r^2 - n - 1$) be the vertices of a triangle ABC, where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangles ABC, then α equals

(1) 6 (2) 18

(3) 8 (4) 20

Answer (4)

Sol. ${}^nC_{r-1} = 28$

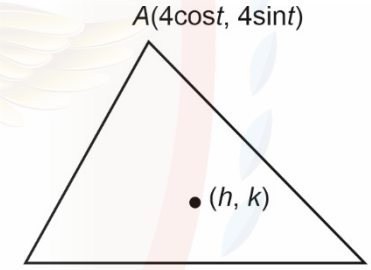
${}^nC_r = 56$

${}^nC_{r+1} = 70$

$\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{28}{56} \Rightarrow \dfrac{r}{n - r + 1} = \dfrac{1}{2}$

$\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{56}{70} \Rightarrow \dfrac{r + 1}{n - r} = \dfrac{70}{56}$

$\Rightarrow n = 8$, $r = 3$



$h = \dfrac{4\cos t + 2\sin t + 1}{3}$ $\quad k = \dfrac{4\sin t - 2\cos t}{3}$

$3h - 1 = 4\cos t + 2\sin t$ …(1)

$3k - 1 = 4\sin t - 2\cos t$ …(2)

$(1)^2 + (2)^2$

$(3h - 1)^2 + (3k)^2 = 20$

Locus of centroid: $(3x - 1)^2 + (3y)^2 = 20$

$\Rightarrow \alpha = 20$

6. Let T_r be ther^{th} term of an A.P. If for some m, $T_m = \dfrac{1}{25}$, $T_{25} = \dfrac{1}{20}$, and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ is equal to

(1) 142 (2) 126

(3) 112 (4) 98

Answer (2)

Sol. $20\sum_{r=1}^{25} T_r = 20\left[\frac{25}{2}\left[a+\frac{1}{20}\right]\right] = 13$

$\Rightarrow \quad a = \frac{1}{20\times 25}$

$\because \quad T_{25} = a + 24d = \frac{1}{20}$

$\Rightarrow \quad d = \frac{1}{20\times 25}$

$T_m = a + (m-1)d = \frac{1}{25}$

$= \frac{1}{20\times 25}m = \frac{1}{25} \Rightarrow \boxed{m = 20}$

Now, $5m\sum_{r=m}^{2m} T_r = 5\times 20\left[\sum_{r=20}^{40} T_r\right]$

$= 100\left[\frac{40}{2}(2a+39d) - \frac{19}{2}(2a+18d)\right]$

$\because \quad a = d$

$\Rightarrow \quad 100\left[\frac{40}{2}\times 41d - \frac{19}{2}\times 21d\right]$

$= 126$

7. Let O be the origin, the point A be $z_1 = \sqrt{3}+2\sqrt{2}i$, the point $B(z_2)$ be such that $\sqrt{3}|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then

(1) ABO is a scalene triangle

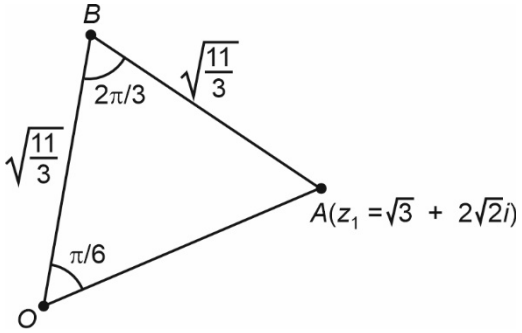
(2) ABO is an obtuse angled isosceles triangle

(3) area of triangle ABO is $\frac{11}{\sqrt{3}}$

(4) area of triangle ABO is $\frac{11}{4}$

Answer (2)

Sol.



$OA = |z_1| = \sqrt{3+8} = \sqrt{11}$

and $OB = \frac{1}{\sqrt{3}}|z_1| = \sqrt{\frac{11}{3}}$

$AB^2 = OA^2 + OB^2 - 2\cdot OA\cdot OB\cos\frac{\pi}{6}$

$= 11 + \frac{11}{3} - 2\cdot\frac{11}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2}$

$\therefore \quad AB = \sqrt{\frac{11}{3}}$

$\therefore$ Area of $\Delta ABD = \frac{1}{2}\cdot OA\cdot OB\cdot\sin\frac{\pi}{6}$

$= \frac{11}{4\sqrt{3}}$ sq. units

Here $OB = AB$ and $\angle A = \frac{2\pi}{3}$

$\therefore \quad \Delta ABD$ is an obtuse angled isosceles triangle.

8. Three defective oranges are accidently mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denotes the number of defective oranges, then the variance of $x$is

(1) 18/25 (2) 26/75

(3) 28/75 (4) 14/25

Answer (3)

Sol. There are 3 bad oranges and 7 good oranges.

$\therefore \quad X$ = number of bad oranges drawn.

X	0	1	2
$P(X)$	$\frac{^7C_2}{^{10}C_2}$	$\frac{^3C_1\times{}^7C_1}{^{10}C_2}$	$\frac{^3C_2}{^{10}C_2}$

$\therefore$ Variance

$$= 0^2 \cdot \frac{{}^7C_2}{{}^{10}C_2} + 1^2 \cdot \left(\frac{3\times 7}{{}^{10}C_2}\right) + 2^2\left(\frac{3}{{}^{10}C_2}\right) - \left(0 + 1\cdot\frac{3\times 7}{{}^{10}C_2} + 2\cdot\frac{3}{{}^{10}C_2}\right)^2$$

$$= \frac{28}{75}$$

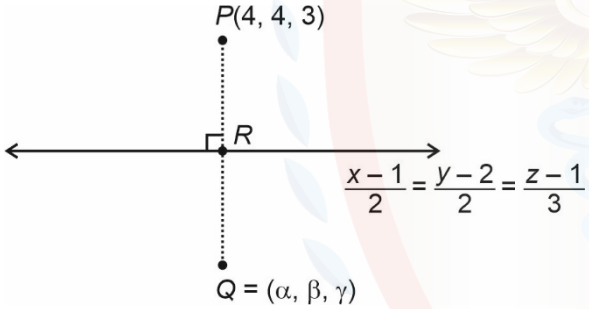
9. If the image of the point (4, 4, 3) in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 12 (2) 7

(3) 9 (4) 8

Answer (3)

Sol.



Let coordinate of $R = (2r + 1, r + 2, 3r + 1)$

$\because$ PR is perpendicular to given line.

$\therefore$ $(2r - 3)\cdot 2 + (r - 2)\cdot 1 + (3r - 2)\cdot 3 = 0$

$\therefore$ $r = 1$

$\therefore$ Coordinate of $R = (3, 3, 4)$

$\therefore$ $(\alpha, \beta, \gamma) = (2, 2, 5)$

$\therefore$ $\alpha + \beta + \gamma = 9$

10. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ is equal to:

(1) 1 (2) $\frac{33}{65}$

(3) $\frac{32}{65}$ (4) 0

Answer (4)

Sol. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\left[\frac{5}{13}\sqrt{1-\frac{33^2}{65^2}} + \frac{33}{65}\sqrt{1-\frac{5^2}{13^2}}\right]\right)$

$$= \cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$$

$$= \cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right)$$

$$= \cos\left(\frac{\pi}{2}\right) = 0$$

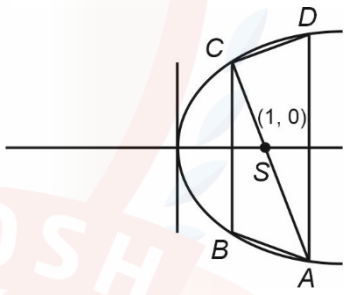
11. Let $ABCD$ be a trapezium whose vertices lie on the parabola $y^2 = 4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point (1, 0), then the area of $ABCD$ is

(1) $\frac{125}{8}$ (2) $\frac{25}{2}$

(3) $\frac{75}{8}$ (4) $\frac{75}{4}$

Answer (4)

Sol.



Take $AS = c$

$$\frac{1}{AS} + \frac{1}{CS} = \frac{1}{a}$$

$$\Rightarrow \frac{1}{c} + \frac{1}{\frac{25}{4} - c} = \frac{1}{a}$$

$$\Rightarrow 4c^2 - 25c + 25 = 0$$

$$\Rightarrow c = \frac{5}{4}, 5$$

$AS = c = 1 + t^2$

$c = \frac{5}{4} \Rightarrow t = \pm\frac{1}{2}$

$c = 5 \Rightarrow t = \pm 2$

$(at^2, 2at) = A\left(\frac{1}{4}, 1\right), D\left(\frac{1}{4}, -1\right), B(4, 4)$

$C(4, -4) \Rightarrow AD = 2,\ BC = 8$

Area of trapezium $= \frac{1}{2}(2+8)\times\frac{15}{4}$

$= \frac{75}{4}$

12. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{(1+e^x)}dx = \pi(\alpha\pi^2+\beta), \alpha,\beta \in \mathbb{Z},$

then $(\alpha + \beta)^2$ equals

(1) 100 (2) 196

(3) 144 (4) 64

Answer (1)

Sol. $I = \int_0^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{1+e^x}dx$

$2I = 2\int_0^{\frac{\pi}{2}} 96x^2\cos^2 x dx$

$I = 96\int_0^{\frac{\pi}{2}} x^2\cos^2 x dx$

$= 48\int_0^{\frac{\pi}{2}} x^2(1+\cos 2x)dx$

$= 2\pi^2 + 48(0-0) - 48\int_0^{\frac{\pi}{2}} x\sin 2x dx$

$= 2\pi^2 - 12\pi + [0-0] = \pi(2\pi^2 - 12)$

$= \pi(\alpha\pi^2 + \beta)$

$\Rightarrow (\alpha + \beta)^2 = 100$

13. The sum of all local minimum values of the function

$$f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7+2|x|), & -1 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$$ is

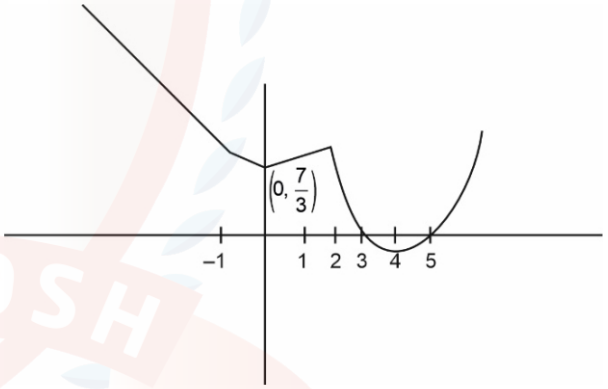
(1) $\frac{131}{72}$ (2) $\frac{167}{72}$

(3) $\frac{171}{72}$ (4) $\frac{157}{72}$

Answer (4)

Sol. $f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7+2|x|), & -1 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7-2x, & -1 \le x < 0 \\ \frac{1}{3}(7+2x), & 0 \le x \le 2 \\ \frac{11}{8}(x-4)(x-5), & x > 2 \end{cases}$$



Minimum value at $x = 0$ and $x = 4.5$

at $x = 0$, $f(0) = \frac{7}{3}$

at $x = 4.5$, $f(4.5) = \frac{-11}{72}$

$\Rightarrow$ Total sum $= \frac{7}{3} - \frac{11}{72}$

$= \frac{157}{72}$

$\Rightarrow$ Option (4) is correct

14. Let $A(x, y, z)$ be a point in xy-plane, which is equidistant from the points (0, 3, 2), (2, 0, 3) and (0,0,1).

Let $B = (1, 4, -1)$ and $C = (2, 0, -2)$. Then among the statements

($S1$) : ΔABC is an isosceles right angled triangle, and

($S2$) : The area of ΔABC is $\frac{9\sqrt{2}}{2}$,

(1) Both are true (2) Only ($S1$) is true

(3) Only ($S2$) is true (4) Both are false

Answer (2)

Sol. $A(x, y, z)$ $P(0, 3, 2)$, $Q(2, 0, 3)$, $R(0, 0. 1)$

$\Downarrow$

As A is in xy-plane

$\Rightarrow$ $A(x, y, 0)$

$AP^2 = AQ^2$

$(x-0)^2 + (y-3)^2 + (0-2)^2 = (x-2)^2 + (y-0)^2 + (0-3)^2$

$x^2 + y^2 - 6y + 9 + 4 = x^2 - 4x + 4 + y^2 + 9$

$\Rightarrow -6y = -4x$

$\Rightarrow 2x = 3y$ ... (i)

$AP^2 = AR^2$

$x^2 + y^2 - 6y + 9 + 4 = x^2 + y^2 + 1$

$\Rightarrow y = 2$

$\Rightarrow x = 3$

$A(3, 2, 0)$, $B(1, 4, -1)$, $C(2, 0, -2)$

$\overrightarrow{AB} = -2\hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{BC} = \hat{i} - 4\hat{j} - \hat{k}$

$\overrightarrow{AC} = -\hat{i} - 2\hat{j} - 2\hat{k}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 4 + 2 = 0$

$\Rightarrow \Delta ABC$ is right angle triangle at A

Also, $AB = 3$, $BC = 3\sqrt{2}$, $AC = 3$

$\Rightarrow AB = AC$

$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times AC = \frac{9}{2}$

$\Rightarrow$ only ($S1$) is true

$\Rightarrow$ option (2) is correct

15. The relation $R = \{(x, y) : x, y \in \mathbb{Z}$ and $x + y$ is even$\}$ is:

(1) Reflexive and transitive but not symmetric

(2) Symmetric and transitive but not reflexive

(3) Reflexive and symmetric but not transitive

(4) An equivalence relation

Answer (4)

Sol. For reflexive $(x, x) \in \mathbb{R}, x \in \mathbb{Z}$

$\Rightarrow x + x = 2x \rightarrow$ even

For symmetric of $(x, y) \in \mathbb{R}$ then $(y, x) \in \mathbb{R}$

when $x, y \in \mathbb{Z}$

$x + y \rightarrow$ even

$\Rightarrow y + x \rightarrow$ even

for transitive if $(x, y) \in \mathbb{R} \Rightarrow x + y \rightarrow$ even

$(y, z) \in \mathbb{R} \Rightarrow y + z \rightarrow$ even

$x + 2y + z \rightarrow$ even

$\Rightarrow x + z$ is even

$\Rightarrow (x, z) \in \mathbb{R}$

$\Rightarrow \mathbb{R}$ is an equivalence relation.

$\Rightarrow$ Option (4) is correct.

16. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x + y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5}|f(i)|$ is

(1) 715 (2) 675

(3) 735 (4) 545

Answer (2)

Sol. Put $y = 0$

$f(x) = f(0) + f(x) + 1 - 0$

$f(0) = -1$

$f(0) = 0 + 0 + b$

$\Rightarrow b = -1$

$f(-1 + 1) = f(-1) + f(1) + 1 + \frac{2}{7}$

$f(0) = f(-1) + f(1) + \frac{9}{7}$

$$-1=(2+3a)+\left(\frac{a+2}{a-1}\right)(-1)+b+(2+3a)$$

$$+\frac{a+2}{a-1}+b+\frac{9}{7}$$

$$-1=4+6a-2+\frac{9}{7}$$

$$-1=2+\frac{9}{7}+6a$$

$$6a=-1-2-\frac{9}{7}$$

$$a=\frac{-5}{7}$$

$$f(x)=\frac{-x^2}{7}+\frac{\frac{9}{7}}{\frac{-12}{7}}x-1$$

$$f(x)=\frac{-x^2}{7}-\frac{3}{4}x-1$$

$$\sum_{i=1}^{5}f(i)=-\frac{1}{7}\left(\frac{5\times6\times11}{6}\right)-\frac{3}{4}\left(\frac{5\times6}{2}\right)-5$$

$$=\frac{-55}{7}-\frac{45}{4}-5$$

$$=\frac{675}{28}$$

$$\Rightarrow 28\left|\sum_{i=1}^{5}f(i)\right|=675$$

17. The area (in sq. units) of the region

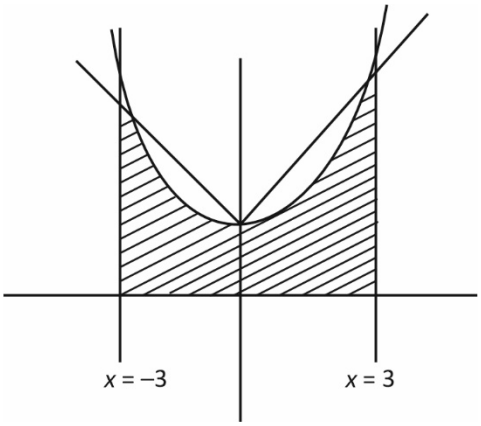
$\{(x, y) : 0 \le y \le 2|x| + 1, 0 \le y \le x^2 + 1, |x| \le 3\}$ is

(1) $\frac{17}{3}$ (2) $\frac{80}{3}$

(3) $\frac{64}{3}$ (4) $\frac{32}{3}$

Answer (3)

Sol.



$$\text{Area}=2\left[\int_0^2(x^2+1)dx+\frac{1}{2}[5+7]\times 1\right]$$

$$=\frac{64}{3}$$

18. If $f(x)=\frac{2^x}{2^x+\sqrt{2}}, x\in\mathbb{R}$, then

$\sum_{k=1}^{81}f\left(\frac{k}{82}\right)$ is equal to

(1) $81\sqrt{2}$ (2) 82

(3) $\frac{81}{2}$ (4) 41

Answer (3)

Sol. $f(x)=\frac{2^x}{2^x+2^{1/2}}=\frac{2^x}{2^x+\sqrt{2}}$

$$f(1-x)=\frac{2^{1-x}}{2^{1-x}+2^{1/2}}=\frac{\frac{2}{2^x}}{\frac{2}{2^x}+2^{1/2}}=\frac{2}{2+\sqrt{2}\,2^x}$$

$$=\frac{\sqrt{2}}{2^x+\sqrt{2}}$$

$$\Rightarrow\quad f(x)+f(1-x)=\frac{\sqrt{2}+2^x}{\sqrt{2}+2^x}=1$$

$$\Rightarrow\quad \sum_{k=1}^{81}f\left(\frac{k}{82}\right)+f\left(\frac{2}{82}\right)+\left(f\left(\frac{3}{82}\right)\right)+....$$

$$...+f\left(\frac{40}{82}\right)+f\left(\frac{41}{82}\right)+f\left(\frac{42}{82}\right)$$

$$+...+f\left(\frac{79}{82}\right)+f\left(\frac{80}{82}\right)+f\left(\frac{81}{82}\right)$$

$$= \left[f\left(\frac{1}{82}\right) + f\left(\frac{81}{82}\right)\right] + \left[f\left(\frac{2}{82}\right) + f\left(\frac{80}{82}\right)\right] + \ldots + \left[f\left(\frac{40}{82}\right) + f\left(\frac{42}{82}\right) + f\left(\frac{41}{82}\right)\right]$$

$$= \underbrace{1+1+\ldots.+1}_{40 \text{ times}} + f\left(\frac{1}{2}\right)$$

$$= 40 + \frac{\sqrt{2}}{\sqrt{2}+\sqrt{2}} = 40 + \frac{1}{2} = \frac{81}{2}$$

19. Let for some function $y = f(x)$, $\int_0^x tf(t)dt = x^2 f(x), x > 0$ and $f(2) = 3$. Then $f(6)$ is equal to

(1) 6 (2) 3

(3) 1 (4) 2

Answer (3)

Sol. $\int_0^x tf(t)dt = x^2 f(x)$

$\Rightarrow xf(x) = 2xf(x) + x^2 f'(x)$

$\Rightarrow x^2 . f'(x) = -x f(x)$

$\Rightarrow \frac{f'(x)}{f(x)} = -\frac{1}{x}$

$\int \frac{f'(x)}{f(x)dx} = -\int \frac{1}{x}dx$

$\Rightarrow \ln f(x) = -\ln x + C$

$f(2) = 3 \Rightarrow \ln 3 = -\ln 2 + C$

$C = \ln 6$

$\Rightarrow f(x) = \frac{6}{x} \Rightarrow f(6) = 1$

20. The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is

(1) $3(2-\sqrt{2})$ (2) $6(3-\sqrt{2})$

(3) $3(3-\sqrt{2})$ (4) $6(2-\sqrt{2})$

Answer (4)

Sol. For $x \geq \frac{3}{2}$

$x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$x = \frac{-2 \pm \sqrt{4+28}}{2} = -1 \pm 2\sqrt{2}$

Only $2\sqrt{2} - 1$ is acceptable root

For $x < \frac{3}{2}$ $x^2 - 2x + 3 - 4 = 0$

$x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

Only $1 - \sqrt{2}$ is acceptable

Sum of the square = $\left(1-\sqrt{2}\right)^2 + \left(2\sqrt{2}-1\right)^2$

$= 6\left(2-\sqrt{2}\right)$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded off to the nearest integer.

21. If $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \, {}^{12}C_{2r-1}$, then the distance of the point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is _____.

Answer (5)

Sol. $\alpha = 1 + \sum_{r=1}^{6} (-1)^{r-1} \, {}^{12}C_{2r-1} 3^{r-1}$

Let $\sqrt{3}i = x$

$\alpha = 1 + \sum_{r=1}^{6} \left[\left(\sqrt{3}i\right)^2\right]^{r-1} \cdot {}^{12}C_{2r-1}$

$\alpha = 1 + \sum_{r=1}^{6} {}^{12}C_{r-1} \frac{\left(\sqrt{3}i\right)^{2r-1}}{\sqrt{3}i}$

$\alpha = 1 + \frac{1}{\sqrt{3}i}\left({}^{12}C_1 x + {}^{12}C_3 x^3 + \ldots {}^{12}C_{11} x^{11}\right)$

$= 1 + \frac{1}{\sqrt{3}i}\left(\frac{\left(1+\sqrt{3}i\right)^{12} - \left(1-\sqrt{3}i\right)^{12}}{2}\right)$

$= 1 + \frac{1}{\sqrt{3}i}\left(\frac{\left(-2\omega^2\right)^{12} - (2\omega)^{12}}{2}\right) = 1$

So, distance of $\left(12, \sqrt{3}\right)$ from $x - \sqrt{3}y + 1 = 0$ is

$\frac{12 - 3 + 1}{2} = 5$

22. Let M denote the set of all real matrices of order 3×3 and let $S = \{-3, -2, -1, 1, 2\}$. Let

$S_1 = \{A = [a_{ij}] \in M : A = A^T \text{ and } a_{ij} \in S, \forall\, i, j\}$,

$S_2 = \{A = [a_{ij}] \in M : A = -A^T \text{ and } a_{ij} \in S, \forall\, i, j\}$,

$S_3 = \{A = [a_{ij}] \in M : a_{11} + a_{22} + a_{33} = 0 \text{ and } a_{ij} \in S, \forall\, i, j\}$.

If $n(S_1 \cup S_2 \cup S_3) = 125\,\alpha$, then α equals _____.

Answer (1613)

Sol. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Number of elements is $S_1 : A = A^T \Rightarrow 5^3 \times 5^3$

Number of elements in $A = -A^T \Rightarrow 0$

Similarly, number of elements in $S_3 \Rightarrow$

$$\left.\begin{array}{lll} a_{11}+a_{22}+a_{33}=0 & \Rightarrow (1,2,-3) \Rightarrow 31 & \\ \text{or } (1,1,-2) & \Rightarrow \quad 3 & \\ \text{or } (-1,-1,2) & \Rightarrow \quad 3 & \end{array}\right\} = 12 \times 5^6$$

$n(S_1 \cap S_3) = 12 \times 5^3$

$n(S_1 \cup S_2 \cup S_3) = 5^6(1+12) - 12 \times 5^3$

$\Rightarrow \quad 5^3 \times [13 \times 5^3 - 12] = 125\alpha$

$\alpha = 1613$

23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{d} = \vec{a} \times \vec{b}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - 2\vec{a}|^2 = 8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$ is equal to _____.

Answer (6)

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{d} = \vec{a} \times \vec{b}$

$= -\hat{i} + \hat{j}$

$|\vec{c} - 2\vec{a}|^2 = 8$

$|\vec{c}|^2 + 4|\vec{a}|^2 - 4\vec{a} \cdot \vec{c} = 8$

$|\vec{c}|^2 + 12 - 4|\vec{c}| = 8$

$|\vec{c}|^2 - 4|\vec{c}| + 4 = 0$

$|\vec{c}|^2 = 2$

$\vec{d} = \vec{a} \times \vec{b}$

$\vec{d} \times \vec{c} = (\vec{a} \times \vec{b}) \times \vec{c}$

$\left(|\vec{d}| \times |\vec{c}| \sin\frac{\pi}{4}\right)^2 = \left((\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}\right)^2$

$4 = 4|\vec{b}|^2 + (\vec{b}.\vec{c})\,2(|\vec{a}|^2) - 2(\vec{b}.\vec{c})(\vec{a}.\vec{b})$

Let $\vec{b}.\vec{c} = x$

$4 = 36 + 3x^2 - 20x$

$3x^2 - 20x + 32 = 0$

$x = \frac{8}{3}, 4$

$\Rightarrow \quad \vec{b}.\vec{c} = \frac{8}{3}, 4$

$\Rightarrow \quad \vec{b}.\vec{c} = \frac{8}{3}$

Now, $|10 - 3\vec{b}.\vec{c}| + |\vec{d} \times \vec{c}|^2$

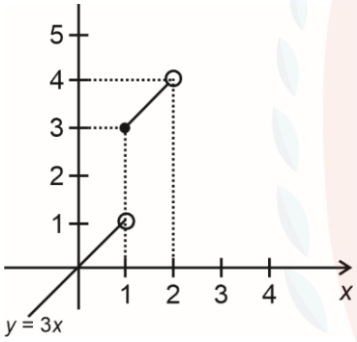
$= |10 - 8| + (2)^2 = 6$

24. Let $f(x)=\begin{cases} 3x & , x<0 \\ \min\{1+x[x],\ x+2[x]\} & , 0\le x\le 2 \\ 5 & , x>2 \end{cases}$

where [.] denotes greatest integer function. If α and β are the number of points, where is not continuous and is not differentiable, respectively, then $\alpha+\beta$ equals _____.

Answer (5)

Sol. $f(x)=\begin{cases} 3x & ;x<0 \\ \min\{1+x,x\} & ;0\le x<1 \\ \min\{2+x,x+2\} & ;1\le x<2 \\ 5 & ;x>2 \end{cases}$



$f(x)=\begin{cases} 3x & ;x<0 \\ x & ;0\le x<1 \\ x+2 & ;1\le x<2 \\ 5 & ;x>2 \end{cases}$

Not continuous at $x\in\{1,2\}\Rightarrow\alpha=2$

Not differentiable at $x\in\{0,1,2\}\Rightarrow\beta=3$

$\therefore\ \alpha+\beta=5$

25. Let $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$ be an ellipse. Ellipses $E_i's$ is constructed such that their centres and eccentricities are same as that of E_1, and length of minor axis of E_i is the length of major axis of $E_{i+1}(i\ge 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty}A_i\right)$, is equal to _____.

Answer (54)

Sol. $E_i:\frac{x^2}{9}+\frac{y^2}{4}=1$

Let b_i be minor axis of E_i and a_i be major axis of E_i

$\Rightarrow\quad e_i=1-\frac{b_i^2}{a_i^2}$

Now, b_{i+1} be minor axis of E_{i+1} and a_{i+1} be major axis of E_{i+1}

$\Rightarrow\quad e_{i+1}=1-\frac{b_{i+1}^2}{a_{i+1}^2}$

Also $a_{i+1}=b_i$ and $e_i=e_{i+1}$

$\Rightarrow\quad \frac{b_i^2}{a_i^2}=\frac{b_{i+1}^2}{b_i^2}$

$\Rightarrow\quad b_{i+1}=\frac{b_i^2}{a_i}$

$\Rightarrow$ Area of $E_i=S_i=\pi a_i b_i$

$\Rightarrow\quad S_{i+1}=\pi a_{i+1}b_{i+1}$

$=\pi(b_i)\left(\frac{b_i^2}{a_i}\right)$

$=\pi(b_i a_i)\left(\frac{b_i}{a_i}\right)^2$

$S_{i+1}=S_i(1-e_i^2)$

$S_{i+1}=S_i\left(1-\left(1-\frac{4}{9}\right)\right)=S_i\left(\frac{4}{9}\right)$

$\Rightarrow\quad S_1=6\pi,\ S_2=6\pi\left(\frac{4}{9}\right),\ S_3=6\pi\left(\frac{4}{9}\right)^2$

$\sum_{k=1}^{\infty}S_i=\left(\frac{6\pi}{1-\frac{4}{9}}\right)=\frac{54\pi}{5}$

$\Rightarrow\quad \frac{5}{\pi}\sum_{k=1}^{\infty}S_i=\frac{5}{\pi}\cdot\frac{54\pi}{5}=54$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. Three infinitely long wires with linear charge density λ are placed along the *x*-axis, *y*-axis and *z*-axis respectively. Which of the following denotes an equipotential surface?

(1) $xy + yz + zx$ = constant

(2) xyz = constant

(3) $(x^2 + y^2)(y^2 + z^2)(z^2 + x^2)$ = constant

(4) $(x + y)(y + z)(z + x)$ = constant

Answer (3)

Sol. Potential due to an infinite wire is $V = 2k\lambda \ln r$, where r is the distance from the wire.

Taking the point in space $P(x, y, z)$

Distance from wire along *x*-axis is $r_x = \sqrt{y^2 + z^2}$

Distance from wire along *y*-axis is $r_y = \sqrt{x^2 + z^2}$

Distance from wire along *z*-axis is $r_z = \sqrt{x^2 + y^2}$

$\Rightarrow$ Potential at *P* due to wire along *x*-axis is

$V_x = 2k\lambda \ln r$

Potential at *P* due to wire along *y*-axis is

$V_y = 2k\lambda \ln r$

Potential at *P* due to wire along *z*-axis is

$V_z = 2k\lambda \ln r$

$\Rightarrow$ Not potential at $P = V = V_x + V_y + V_z$

or $V = 2k\lambda \ln r_x + 2k\lambda \ln r_y + 2k\lambda \ln r_z$

i.e. $V = 2k\lambda \ln(r_x r_y r_z)$

or $V = 2k\lambda \ln\left(\sqrt{y^2 + z^2}\sqrt{z^2 + x^2}\sqrt{x^2 + y^2}\right)$

$= k\lambda \ln(y^2 + z^2)(z^2 + x^2)(x^2 + y^2)$

$\Rightarrow$ For equipotential surface

$(x^2 + y^2)(y^2 + z^2)(z^2 + x^2)$ = constant

27. Choose the correct nuclear process from the below options

[*p*: proton, *n*: neutron, e^-: electron, e^+: positron, ν : neutrino, $\bar{\nu}$: antineutrino]

(1) $n \to p + e^+ + \nu$ (2) $n \to p + e^+ + \bar{\nu}$

(3) $n \to p + e^- + \bar{\nu}$ (4) $n \to p + e^- + \nu$

Answer (3)

Sol. For all nuclear processes, charge must be conserved. Also, when a release of an electron (e^-) is always accompanied by a release of an antineutrino ($\bar{\nu}$)

Hence, $n \to p + e^- + \bar{\nu}$ is the correct answer.

28. Consider a long thin conducting wire carrying a uniform current *I*. A particle having mass "*M*" and charge "*q*" is released at a distance "*a*" from the wire with a speed v_0 along the direction of current in the wire. The particle gets attracted to the wire due to magnetic force. The particle turns round when it is at distance *x* from the wire. The value of *x* is

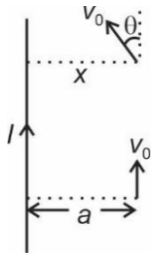
[μ_0 is vacuum permeability]

(1) $a\left[1 - \frac{mv_0}{q\mu_0 I}\right]$ (2) $\frac{a}{2}$

(3) $a\left[1 - \frac{mv_0}{2q\mu_0 I}\right]$ (4) $ae^{\frac{-4\pi mv_0}{q\mu_0 I}}$

Answer (4)

Sol. Let the velocity of the particle make an angle θ with initial direction when it is at a distance x.



$$\Rightarrow \quad \frac{dx}{dt} = -v_\theta \sin \qquad \ldots(i)$$

Also, $\frac{d\theta}{dt} = \frac{q}{m}\left(\frac{\mu_0 I}{2\pi x}\right)$ $\qquad \ldots(ii)$

$$\Rightarrow \quad \frac{dx}{d\theta} = \frac{-2\pi m v_\theta x \sin}{\mu \quad q \quad _0 I} \quad \ldots \text{dividing eq(i) with eq(ii)}$$

or $\int_a^x \frac{dx}{x} = \frac{-2\pi m v_0}{q\mu_0 I}\int_0^\pi \sin\theta\, d\theta$

$$\ln\frac{x}{a} = \frac{-4\pi m v_0}{q\mu_0 I}$$

Or $x = ae^{\frac{-4\pi m v_0}{q\mu_0}}$

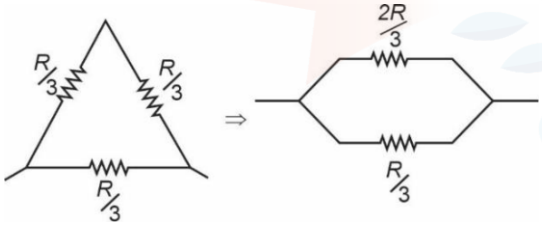
29. A wire of resistance R is bent into an equilateral triangle and an identical wire is bent into a square. The ratio of resistance between the two end points of an edge of the triangle to that of the square is

(1) 32/27 (2) 27/32

(3) 8/9 (4) 9/8

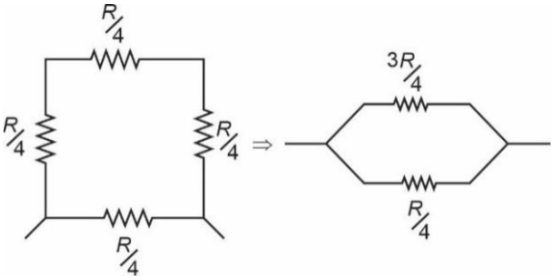
Answer (1)

Sol. For the wire bent into an equilateral triangle, each side has a resistance $\frac{R}{3}$.



$$R_{eq} = \frac{\left(\frac{2R}{3}\right)\left(\frac{R}{3}\right)}{\frac{2R}{3}+\frac{R}{3}} = \frac{2R}{9} = \ _1 \text{ (lets say)}$$

For the wire bent into a square, each side has a resistance $\frac{R}{4}$.



$$R_{eq} = \frac{\left(\frac{3R}{4}\right)\left(\frac{R}{4}\right)}{\frac{3R}{4}+\frac{R}{4}} = \frac{3R}{16} = \ _3 \text{ (lets say)}$$

$$\Rightarrow \quad \frac{R_1}{R_3} = \frac{\frac{2R}{9}}{\frac{3R}{16}} = \frac{32}{27}$$

30. A proton of mass 'm_p' has same energy as that of a photon of wavelength 'λ'. If the proton is moving at non-relativistic speed, then ratio of its de Broglie wavelength to the wavelength of photon is.

(1) $\frac{1}{2c}\sqrt{\frac{E}{m_p}}$ (2) $\frac{1}{c}\sqrt{\frac{E}{m_p}}$

(3) $\frac{1}{c}\sqrt{\frac{E}{2m_p}}$ (4) $\frac{1}{c}\sqrt{\frac{2E}{m_p}}$

Answer (3)

Sol. Energy of photon = $E = \frac{hc}{\lambda}$

$\Rightarrow$ Wavelength of photon = $\lambda = \frac{hc}{E}$

Energy of proton = $E = \frac{1}{2}m_p v^2 \quad \frac{P^2}{m_p}$

$\Rightarrow$ Linear momentum of proton = $P = \sqrt{2m_p E}$

Or de-Broglie wavelength of proton

$$= \lambda_p = \frac{h}{P} = \frac{h}{\sqrt{2m_p E}}$$

Ratio $\frac{\lambda_p}{\lambda} = \frac{h \quad E}{\sqrt{2m_p E}} \quad \frac{}{hc}$

$$= \frac{1}{c}\sqrt{\frac{E}{2m_p}}$$

31. A Carnot engine (E) is working between two temperatures 473 K and 273 K. In a new system two engines – engine E_1 works between 473 K to 373 K and engine E_2 works between 373 K to 273 K. If η_{12}, η_1 and η_2 are the efficiencies of the engines E, E_1 and E_2 respectively, then

(1) $\eta_{12} = \eta_1 + \eta_2$ (2) $\eta_{12} \geq \eta_1 + \eta_2$

(3) $\eta_{12} < \eta_1 + \eta_2$ (4) $\eta_{12} = \eta_1 \eta_2$

Answer (3)

Sol. Efficiencies of a carnot engine $\eta = 1 - \frac{T_{sink}}{T_{source}}$

$$\Rightarrow \quad \eta_1 = 1 - \frac{373\,K}{473\,K} = \frac{100}{473}$$

$$\eta_2 = 1 - \frac{273\,K}{373\,K} = \frac{100}{373}$$

$$\eta_{12} = 1 - \frac{273\,K}{473\,K} = \frac{100}{473}$$

$$\eta_{12} - \eta_1 = \frac{200}{473} - \frac{100}{473} = \frac{100}{473} < \frac{100}{373}$$

$$\Rightarrow \quad \eta_{12} - \eta_1 < \eta_2$$

or $\eta_{12} < \eta_1 + \eta_2$

32. Due to presence of an em-wave whose electric component is given by $E = 100 \sin(\omega t - kx)$ NC^{-1}, a cylinder of length 200 cm holds certain amount of em-energy inside it. If another cylinder of same length but half diameter than previous one holds same amount of em-energy, the magnitude of the electric field of the corresponding em-wave should be modified as

(1) $50 \sin(\omega t - kx)$ NC^{-1} (2) $400 \sin(\omega t - kx)$ NC^{-1}

(3) $25 \sin(\omega t - kx)$ NC^{-1} (4) $200 \sin(\omega t - kx)$ NC^{-1}

Answer (4)

Sol. Energy density of an $EM_{wave} = \frac{1}{2}\varepsilon E_0^2$, where E_0 is the amplitude of the wave.

Since total energy is same for both cylinders

$$\left(\frac{1}{2}\varepsilon E_1^2\right)\pi R_1^2 L_1 = \left(\frac{1}{2}\varepsilon E_2^2\right)\pi R_2^2 L_2$$

$$\Rightarrow \quad E_1^2 R_1^2 L_1 = E_2^2 R_2^2 L_2$$

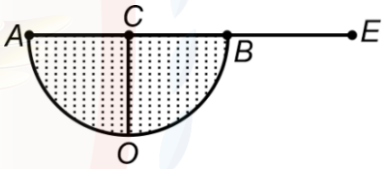
or $E_2 = \frac{E_1 R_1}{R_2}\sqrt{\frac{L_1}{L}} = \frac{100\ d}{(d/2)}\sqrt{\frac{L_1}{L1}} = 200$ N/C

$[\ L_1 L_2 = \quad 200$ cm$]$

$\Rightarrow$ The amplitude of corresponding EM wave is 200 N/C

or the wave is $E = 200 \sin(\omega t - kx)$ NC^{-1}

33. A hemispherical vessel is completely filled with a liquid of refractive index μ. A small coin is kept at the lowest point (O) of the vessel as shown in figure. The minimum value of the refractive index of the liquid so that a person can see the coin from point E (at the level of the vessel) is _____.

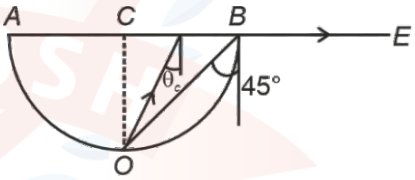


(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

Answer (1)

Sol.



For the rays from coin to reach the point E, the refracted rays must grazing the surface, *i.e.* they must be incident at critical angle θ_c inside the liquid.

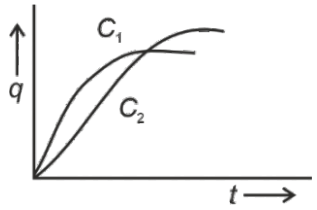
$$\mu = \frac{1}{\sin\theta_c}$$

μ is minimum when θ_c is maximum.

Maximum value of $\theta_c = 45°$

$\Rightarrow$ μ has a minimum value of $\sqrt{2}$.

34. Two capacitors C_1 and C_2 are connected in parallel to a battery. Charge-time graph is shown below for the two capacitors. The energy stored with them are U_1 and U_2, respectively. Which of the given statements is true?



(1) $C_2 > C_1$, $U_2 < U_1$ (2) $C_1 > C_2$, $U_1 > U_2$
(3) $C_1 > C_2$, $U_1 < U_2$ (4) $C_2 > C_1$, $U_2 > U_1$

Answer (4)

Sol. For a capacitor at steady state

$q = CV$ and $U = \frac{1}{2}CV^2$

Since C_1 and C_2 are connected in parallel, $V_1 = V_2$.

Also from graph $q_1 < q_2$

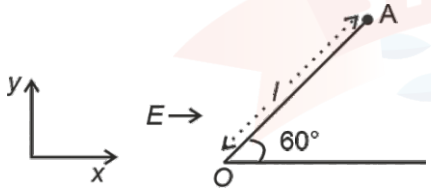
$\Rightarrow C_1V_1 < C_2V_2$

i.e. $C_1 < C_2$ or $C_2 > C_1$

$$\frac{U_1}{U_2} = \frac{C_1V_1^2}{C_2V_2^2} = \frac{C_1}{C_2} < 1$$

or $U_1 < U_2$ or $U_2 > U_1$

35. A particle of mass 'm' and charge 'q' is fastened to one end 'A' of a massless string having equilibrium length l, whose other end is fixed at point 'O'. The whole system is placed on a frictionless horizontal plane and is initially at rest. If uniform electric field is switched on along the direction as shown in figure, then the speed of the particle when it crosses the x-axis is



(1) $\sqrt{\frac{qEl}{m}}$ (2) $\sqrt{\frac{qEl}{4m}}$
(3) $\sqrt{\frac{qEl}{2m}}$ (4) $\sqrt{\frac{2qEl}{2m}}$

Answer (1)

Sol. From work energy theorem,

Work done by electric force = change in kinetic energy

or $qE\left(l - \frac{l}{2}\cos 60^\circ\right) = \frac{1}{2}mv^2$

$\Rightarrow \frac{qEl}{2} = \frac{1}{2}mv^2$

or $v = \sqrt{\frac{qEl}{m}}$

36. In the experiment for measurement of viscosity 'η' of given liquid with a ball having radius R, consider following statements.

A. Graph between terminal velocity V and R will be a parabola.
B. The terminal velocities of different diameter balls are constant for a given liquid.
C. Measurement of terminal velocity is dependent on the temperature.
D. This experiment can be utilized to assess the density of a given liquid.
E. If balls are dropped with some initial speed, the value of η will change.

Choose the correct answer from the options given below:

(1) B, D and E only (2) A, C and D only
(3) A, B and E only (4) C, D and E only

Answer (2)

Sol. The terminal velocity of ball of radius R inside a liquid of viscosity η can be written as $V_T = \frac{2R^2g}{n}(\sigma - \rho)$, where σ is the density of ball and δ is the density of the liquid.

Hence,

A is correct since $V_T \alpha R^2$ gives a parabola on a graph

C is correct since $V_T \alpha \frac{1}{\eta}$ and η varies with temperature

D is correct since $V_T \alpha (\sigma - \rho)$ *i.e.*, varies with density of liquid.

37 Consider following statements:

A. Surface tension arises due to extra energy of the molecules at the interior as compared to the molecules at the surface, of a liquid.

B. As the temperature of liquid rises, the coefficient of viscosity increases.

C. As the temperature of gas increases, the coefficient of viscosity increases.

D. The onset of turbulence is determined by Reynold's number.

E. In a steady flow two stream lines never intersect.

Choose the correct answer from the options given below:

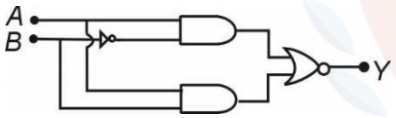
(1) A, D, E only (2) C, D, E only

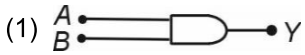
(3) A, B, C only (4) B, C, D only

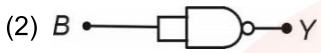
Answer (2)

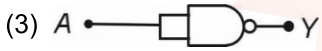
Sol. Surface tension arises due to extra energy of the molecules at the molecules at the surface as compared at the interior of a liquid. The coefficient of viscosity for a liquid decreases with rise in temperature where as it increases for gases with increase in temperature. The flow is turbulent for a Reynold's number greater than 2000. Stream lines never intersect in a steady flow.

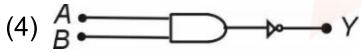
38. Which of the following circuits has the same output as that of the given circuit?



(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. $Y = \overline{A\bar{B} \quad AB}$

$= \overline{A\left(\bar{B} \quad B\right)}$

$= \bar{A}$ as $\left(\bar{B} + B\right) = 1$

i.e. NOT gate with *A* as input

39. The centre of mass of a thin rectangular plate (fig – x) with sides of length *a* and *b*, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be _____.

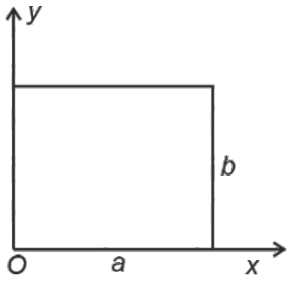


fig - x

(1) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (2) $\left(\frac{1}{3}a, \frac{b}{2}\right)$

(3) $\left(\frac{2}{3}a, \frac{2}{3}b\right)$ (4) $\left(\frac{2}{3}a, \frac{b}{2}\right)$

Answer (4)

Sol. $dm = \sigma\, dA$

$= \sigma(dx)(dy) = \frac{\sigma_0 x}{ab}(dx)(dy)$

$$x_{com} = \frac{\int x dm}{\int dm} = \frac{\int x\frac{(\sigma_0 x)}{ab}(dx)(dy)}{\int \frac{\sigma_0 x}{ab}(dx)(dy)}$$

$$= \frac{\int_0^a x^2 dx \int_0^b dy}{\int_0^a x dx \int_0^b dy} = \frac{2a}{3}$$

$$y_{com} = \frac{\int y dm}{\int dm} = \frac{\int y\left(\frac{\sigma_0 x}{ab}\right)(dx)(dy)}{\int \frac{\sigma_0 x}{ab}(dx)(dy)}$$

$$= \frac{\int_0^a x dx \int_0^b y dy}{\int_0^a x dx \int_0^b dy} = \frac{b}{2}$$

i.e., $r_{com} \equiv \left(\frac{2a}{3}, \frac{b}{2}\right)$

40. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: A sound wave has higher speed in solids than gases.

Reason R: Gases have higher value of Bulk modulus than solids.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) **A** is true but **R** is false

(2) **A** is false but **R** is true

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

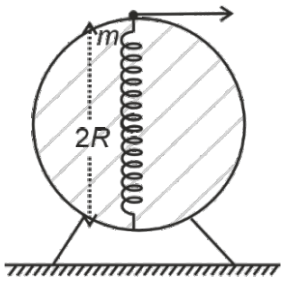
(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

Answer (1)

Sol. Speed of sound in a medium depends on inertial and elastic properties as $v = \sqrt{\frac{B}{P}}$ for gases and $v = \sqrt{\frac{Y}{P}}$ for solids. Since the elastic property of solid happens to be many folds greater than that of gases, the speed of sound in solids i5 higher than in gases.

Also, bulk modulus of gases varies between 0 and $\infty\left(B = -v\frac{dP}{dv}\right)$ hence reason is false.

41. A bead of mass 'm' slides without friction on the wall of a vertical circular hoop of radius 'R' as shown in figure. The bead moves under the combined action of gravity and a massless spring (k) attached to the bottom of the hoop. The equilibrium length of the spring is 'R'. If the bead is released from top of the hoop with (negligible) zero initial speed, velocity of bead, when the length of spring becomes 'R', would be (spring constant is 'k', g is acceleration due to gravity)

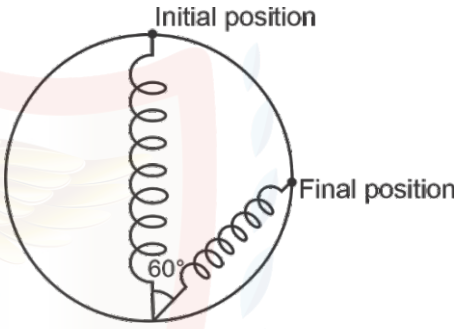


(1) $\sqrt{2Rg + \frac{4kR^2}{m}}$ (2) $\sqrt{2Rg + \frac{kR^2}{m}}$

(3) $\sqrt{3Rg + \frac{kR^2}{m}}$ (4) $2\sqrt{gR + \frac{kR^2}{m}}$

Answer (3)

Sol.



Work done by gravity = $mg(2R - R\cos 60°)$

$= \frac{3mgR}{2}$

Work done by spring $= -\frac{1}{2}k\left(0^2 - R^2\right)$

$= \frac{1}{2}kR^2$

Net work = change in kinetic energy

i.e. $\frac{3mgR}{2} + \frac{kR^2}{2} = \frac{1}{2}mv^2$

or $v^2 = 3gR + \frac{kR^2}{m}$

or $v = \sqrt{3gR + \frac{kR^2}{m}}$

42. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : In a central force field, the work done is independent of the path chosen.

Reason R : Every force encountered in mechanics does not have not have an associated potential energy.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true but **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

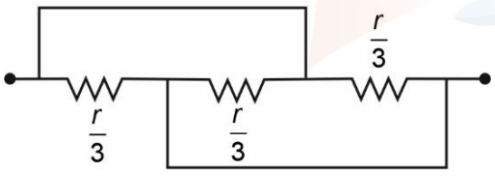
Answer (1)

Sol. Assertion is correct as central forces are conservative in nature, *i.e.* work done is independent of path.

Reason is true as some forces in mechanics like, friction are non-conservative because work done depends on path and only conservative forces have an associated potential energy.

Also, reason does not explain assertion.

43. Find the equivalent resistance between two ends of the following circuit



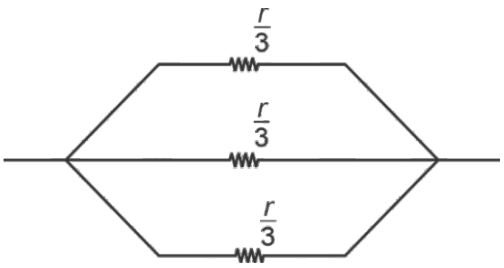
(1) $\frac{r}{6}$ (2) $\frac{r}{9}$

(3) r (4) $\frac{r}{3}$

Answer (2)

Sol. The three resistors are in parallel

i.e.



$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{\frac{r}{3}} + \frac{1}{\frac{r}{3}} + \frac{1}{\frac{r}{3}}$$

Or $R_{eq} = \frac{r}{9}$

44. A thin prism P_1 with angle 4° made of glass having refractive index 1.54, is combined with another thin prism P_2 made of glass having refractive index 1.72 to get dispersion without deviation. The angle of the prism P_2 is degrees is

(1) $\frac{16}{3}$

(2) 1.5

(3) 4

(4) 3

Answer (4)

Sol. For dispersion without deviation,

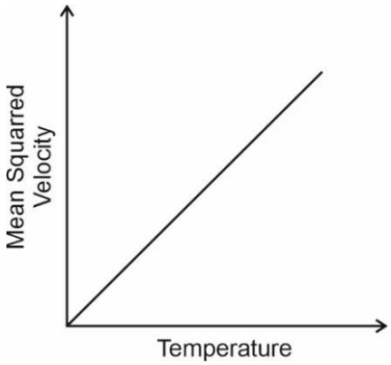
$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$

$\Rightarrow (1.54 - 1)4° = (1.72 - 1)A_2$

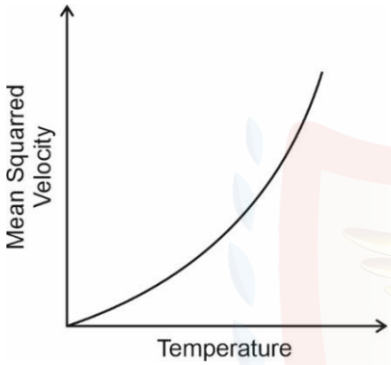
Or $A_2 = \frac{0.54}{0.72} \times 4 = 3°$

45. For a particular ideal gas which of the following graphs represents the variation of mean squared velocity of the gas molecules with temperature?

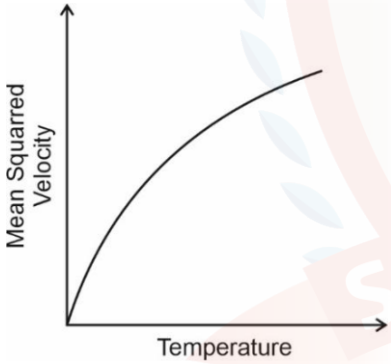
(1)



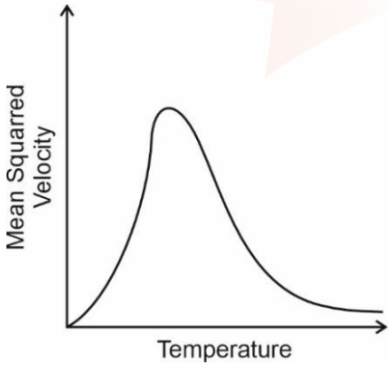
(2)



(3)



(4)



Answer (1)

Sol. Mean squared velocity $= \left(v_{\text{rms}}\right)^2 \quad \frac{3RT}{M}$

i.e. (mean squared velocity) $\propto$ (Temperature)

Hence, graph is a straight line.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A double slit interference experiment performed with a light of wavelength 600 nm forms an interference fringe pattern on a screen with 10^{th} bright fringe having its centre at a distance of 10 mm from the central maximum. Distance of the centre of the same 10^{th} bright fringe from the central maximum when the source of light is replaced by another source of wavelength 660 nm would be _____ mm.

Answer (11)

Sol. Position of the n^{th} bright fringe w.r.t. central maxima in a YDSE is $y_n = \frac{\lambda D}{d}$.

$$\Rightarrow \quad \frac{y'_{10}}{y_{10}} = \frac{\lambda'}{\lambda}$$

or $$y'_{10} = \frac{\lambda'}{\lambda} y_{10}$$

$$= \left(\frac{660 \text{ nm}}{600 \text{ nm}}\right) 10 \text{ mm}$$

$$= 11 \text{ mm}$$

47. In a measurement, it is asked to find modulus of elasticity per unit torque applied on the system. The measured quantity has dimension of $[M^aL^bT^c]$. If $b = 3$, then value of c is _____.

Answer (0)

Sol. [Modulus of elasticity] = $ML^{-1}T^{-2}$

[Torque] = ML^2T^{-2}

[Modulus of elasticity per unit torque]

$$= \frac{ML^{-1}T^{2}}{ML^{2}T^{-2}} \quad L^{-3}$$

48. Two iron solid discs of negligible thickness have radii R_1 and R_2 and moment of inertia I_1 and I_2, respectively. For $R_2 = 2R_1$, then ratio of I_1 and I_2 would be $1/x$, where x = _____.

Answer (16)

Sol. Given $\frac{M_1}{M_2} = \frac{\pi R_1^2}{\pi R_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{1}{4}$

$$\frac{I_1}{I_2} = \frac{M_1}{M_2} \times \left(\frac{R_1}{R_2}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = \frac{1}{x}$$

$x = 16$

49. A tiny metallic rectangular sheet has length and breadth of 5 mm and 2.5 mm, respectively. Using a specially designed screw gauge which has pitch of 0.75 mm and 15 divisions in the circular scale, you are asked to find the area of the sheet. In this measurement, the maximum fractional error will be $\frac{x}{100}$ where x is _____.

Answer (3)

Sol. Least count of screw gauge =

$$\frac{\text{Pitch}}{\text{Number of circular scale divisions}}$$

$$\Rightarrow \text{LC} = \frac{0.75 \text{ mm}}{15} \quad 0.05 \text{ mm}$$

Length, ℓ = 5 mm

Breadth, b = 2.5 mm

Area = ℓb

$\Rightarrow$ % error in area = (% error in length) + (% error in breadth) = $\frac{\Delta}{} \times 100 + \frac{b}{b} \times 100$

$$= \frac{0.05}{5} \times 100 + \frac{0.05}{2.5} \times 100$$

= 3%

50. The moment of inertia of a solid disc rotating along its diameter is 2.5 times higher than the moment of inertia of a ring rotating in similar way. The moment of inertia of a solid sphere which has same radius as the disc and rotating in similar way, is n times higher than the moment of inertia of the given ring. Here, n = _____.

Consider all the bodies have equal masses.

Answer (4)

Sol. Given,

$M_{disc} = M_{ring} = M_{sphere} = m$(let's say) and $R_{sphere} = R_{disc}$

$I_{disc} = 2.5\ I_{ring}$

$$\frac{mR_{disc}^2}{4} = 2.5\frac{mR_{ring}^2}{2}$$

$\Rightarrow R_{disc}^2 \quad 5 \quad _{ring}$

$I_{sphere} = nI_{ring}$

$$\frac{2}{5}mR_{disc}^2 = n\frac{mR_{ring}^2}{2}$$

$n = 4$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Match the **LIST-I** with **LIST-II**

	LIST-I (Redox Reaction)		**LIST-II (Type of Redox Reaction)**
A.	$CH_{4(g)} + 2O_{2(g)} \xrightarrow{\Delta} CO_{2(g)} + 2H_2O_{(l)}$	I.	Disproportionation reaction
B.	$2NaH_{(s)} \xrightarrow{\Delta} 2Na_{(s)} + H_{2(g)}$	II.	Combination reaction
C.	$V_2O_{5(s)} + 5Ca_{(s)} \xrightarrow{\Delta} 2V_{(s)} + 5CaO_{(s)}$	III.	Decomposition reaction
D.	$2H_2O_{2(aq)} \xrightarrow{\Delta} 2H_2O_{(l)} + O_{2(g)}$	IV.	Displacement reaction

Choose the ***correct*** answer from the options given below:

(1) A-II, B-III, C-I, D-IV (2) A-IV, B-I, C-II, D-III

(3) A-III, B-IV, C-I, D-II (4) A-II, B-III, C-IV, D-I

Answer (4)

Sol. (A) $CH_4(g) + 2O_2(g) \xrightarrow{\Delta} 2CO_2(g) + H_2O(l) \Rightarrow$ combination reaction

(B) $2NaH(s) \xrightarrow{\Delta} 2Na(s) + H_2(g) \Rightarrow$ Decomposition reaction

(C) $V_2O_5(s) + 5Ca(s) \xrightarrow{\Delta} 2V(s) + 5CaO(s)$ $\Rightarrow$ Displacement reaction

(D) $2H_2O_2$ (aq.) $\xrightarrow{\Delta} 2H_2O(l) + O_2(g)$ $\Rightarrow$ Disproportionation reaction

52. Given below are two statements:

Statement I: D-glucose pentaacetate reacts with 2, 4-dinitrophenylhydrazine

Statement II: Starch, on heating with concentrated sulfuric acid at 100°C and 2-3 atmosphere pressure produces glucose.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Answer (3)

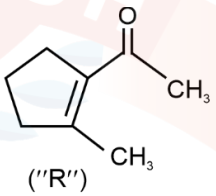
Sol. • D-glucose pentaacetate do not react with 2,4 DNP

• Starch $\xrightarrow[\Delta]{\text{conc.}H_2SO_4}$ Glucose

2 – 3 atm

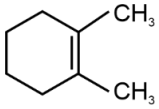
100°C

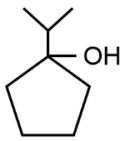
53. A molecule (''P'') on treatment with acid undergoes rearrangement and gives (''Q''). (''Q'') on ozonolysis followed by reflux under alkaline condition gives (''R''). The structure of (''R'') is given below.

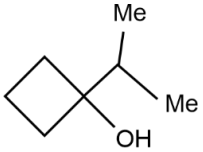


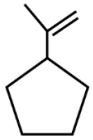
(''R'')

The structure of **(''P'')** is

(1) 

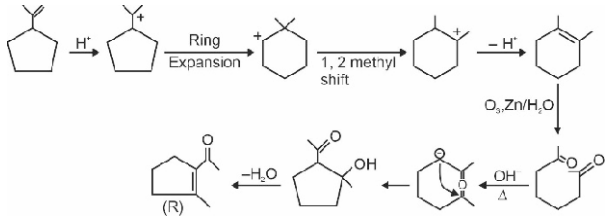
(2) 

(3) 

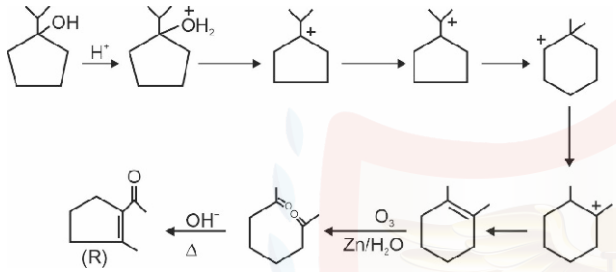
(4) 

Answer (2 and 4)

Sol.



Or



54.

$[A]_0$ / mol L^{-1}	$t_{1/2}$ / min
0.100	200
0.025	100

For a given reaction R → P, $t_{1/2}$ is related to $[A]_0$ as given in table.

Given: log 2 = 0.30

Which of the following is <u>true</u>?

A. The order of the reaction is ½.

B. If $[A]_0$ is 1M, then $t_{1/2}$ is $200\sqrt{10}$ min

C. The order of the reaction changes to 1 if the concentration of reactant changes from 0.100 M to 0.500 M.

D. $t_{1/2}$ is 800 min for $[A]_0$ = 1.6 M

Choose the ***correct*** answer from the options given below:

(1) A, B and D only (2) A and C only

(3) C and D only (4) A and B only

Answer (1)

Sol. $t_{\frac{1}{2}} \propto (C_0)^{1-\eta}$

$$\frac{t_1}{t_2} = \left(\frac{C_1}{C_2}\right)^{1-\eta}$$

$$\Rightarrow \frac{200}{100} = \left(\frac{0.100}{0.025}\right)^{1-\eta}$$

$$\Rightarrow 2 = (4)^{1-\eta}$$

$$(1-\eta) = \frac{1}{2}$$

$$\eta = \frac{1}{2}$$

For $\eta = \frac{1}{2}$

$$\frac{-dA}{dt} = k(A)^{\frac{1}{2}}$$

$$\int_{C_0}^{C} \frac{dA}{(A)^{\frac{1}{2}}} = -\int_0^t kdt$$

$$\Rightarrow 2A^{\frac{1}{2}} = -kt$$

$$\Rightarrow \sqrt{C} - \sqrt{C_0} = \frac{-kt}{2}$$

$$\sqrt{C} - \sqrt{C_0} - \frac{kt}{2}$$

For $C_0 = 0.1 \Rightarrow t_{\frac{1}{2}} = 200$ min

$$\sqrt{\frac{c_0}{2}} = \sqrt{c_0} - \frac{kt}{2}$$

$$\frac{kt}{2} = \sqrt{c_0} - \sqrt{\frac{c_0}{2}}$$

$$\frac{kt}{2} = \sqrt{c_0}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$t_{\frac{1}{2}} = \frac{2\sqrt{c_0}}{k}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$200 = \frac{2\sqrt{0.1}}{k}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$k = \frac{\sqrt{0.1}}{100}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

For C_0 = 1M

$$t_{\frac{1}{2}} = \frac{2\times 100\left(\sqrt{2}\right)}{\sqrt{0.1}\left(\sqrt{2}-1\right)}\times\frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$$

$\Rightarrow 200\sqrt{10}$ min.

$\Rightarrow$ B is correct

C is incorrect

For C_0 = 1.6 M

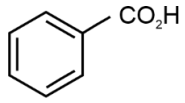
$$t_{\frac{1}{2}} = \frac{2\sqrt{1.6}\left(\sqrt{2}\right)\left(\sqrt{2}-1\right)\times 100}{\sqrt{0.1}\left(\sqrt{2}-1\right)\left(\sqrt{2}\right)}$$

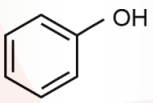
$t_{\frac{1}{2}} = 400\times 2$ min

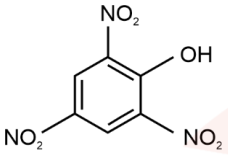
$t_{\frac{1}{2}} = 800$ min

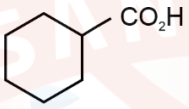
A, B and D are correct

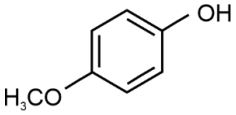
55. The compounds that produce CO_2 with aqueous $NaHCO_3$ solution are:

A. 

B. 

C. 

D. 

E. 

Choose the ***correct*** answer from the options given below:

(1) A, C and D only (2) A and C only

(3) A and B only (4) A, B and E only

Answer (1)

Sol. A, C, D produces CO_2 with $NaHCO_3$ as these are stronger acid as compared to H_2CO_3 (carbonic acid)

56. The incorrect decreasing order of atomic radii is

(1) Si > P > Cl > F (2) Mg > Al > C > O

(3) Be > Mg > Al > Si (4) Al > B > N > F

Answer (3)

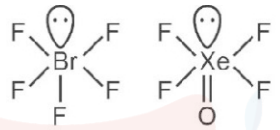
Sol. Correct order or atomic radii Be < Mg > Al > Si.

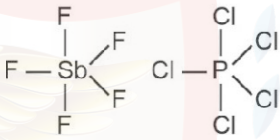
57. The molecules having square pyramidal geometry are

(1) BrF_5 & PCl_5 (2) BrF_5 & $XeOF_4$

(3) SbF_5 & $XeOF_4$ (4) SbF_5 & PCl_5

Answer (2)

Sol.  $\Rightarrow$ Square pyramidal

 $\Rightarrow$ Triangular bipyramidal

58. [illegible] pressure of the system increased 2 times keeping temperature constant, then identify correct observation from following

(1) Volume of system increases

(2) The solid phase (ice) disappears completely

(3) Liquid phase disappears completely

(4) The amount of ice decreases

Answer (2)

Sol.

$$H_2O(s) \rightleftharpoons H_2O(l)$$

On increasing pressure to 2 atm, freezing point of ice will decrease and solid phase (ice) will disappear completely.

59. In a multielectron atom, which of the following orbitals described by three quantum numbers will have same energy in absence of electric and magnetic fields?

A. n = 1, l = 0, m_l = 0

B. n = 2, l = 0, m_l = 0

C. n = 2, l = 1, m_l = 1

D. n = 3, l = 2, m_l = 1

E. n = 3, l = 2, m_l = 0

Choose the **correct** answer from the options given below:

(1) A and B only (2) C and D only

(3) B and C only (4) D and E only

Answer (4)

Sol. (n + l) must be same.

	(n + l)
A	1
B	2
C	3
D	5
E	5

(n + l) value for D and E is same.

60. Which of the following oxidation reactions are carried out by using $K_2Cr_2O_7$ and $KMnO_4$ in acidic medium?

A. $I^- \rightarrow I_2$ B. $S^{2-} \rightarrow S$

C. $Fe^{2+} \rightarrow Fe^{3+}$ D. $I^- \rightarrow IO_3^-$

E. $S_2O_3^{2-} \rightarrow SO_4^{2-}$

Choose the **correct** answer from the option given below.

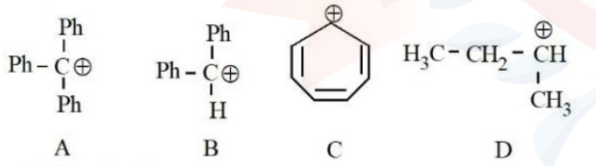
(1) A, B and C only (2) A, D and E only

(3) C, D and E only (4) B, C and D only

Answer (1)

Sol. I^- gets converted in I_2 by acidified $KMnO_4$.

61. The correct order of stability of following carbocations is

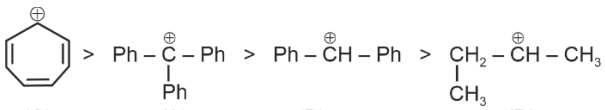


A B C D

(1) A > B > C > D (2) C > B > A > D

(3) C > A > B > D (4) B > C > A > D

Answer (3)

Sol. 

(C) > (A) > (B) > (D)

62. Consider 'n' is the number of lone pair of electrons present in the equatorial position of the most stable structure of ClF_3. The ions from the following with 'n' number of unpaired electrons are

A. V^{3+} B. Ti^{3+}

C. Cu^{2+} D. Ni^{2+}

E. Ti^{2+}

Choose the ***correct*** answer from the options given below:

(1) A, D and E Only (2) B and C Only

(3) A and C Only (4) B and D Only

Answer (1)

Sol. n = 2

V^{3+} : $4s^0 3d^2$ (n = 2) $\Rightarrow$ A is correct

Ti^{3+} : $4s^0 3d^1$ (n = 1) $\Rightarrow$ B is not correct

Cu^{2+} : $4s^0 3d^9$ (n = 1) $\Rightarrow$ C is not correct

Ni^{2+} : $4s^0 3d^8$ (n = 2) $\Rightarrow$ D is correct

Ti^{2+} : $4s^0 3d^2$ (n = 2) $\Rightarrow$ E is correct

A, D and E are correct

Option (1) is correct

63. A weak acid HA has degree of dissociation x. Which option gives the correct expression of (pH – pK_a)?

(1) log(1 + 2x)

(2) $\log\left(\frac{1-x}{x}\right)$

(3) $\log\left(\frac{x}{1-x}\right)$

(4) 0

Answer (3)

Sol. $\underset{C}{H} \rightleftharpoons \underset{-}{H^+} + \underset{-}{A^-}$

$\underset{C(1-x)}{} \quad \underset{Cx}{} \quad \underset{Cx}{}$

$$K_a = \frac{[H^+](x)}{C(1-x)}$$

$$[H^+] = K_a \frac{(1-x)}{x}$$

$$\log H^+ = \log K_a + \log\left(\frac{1-x}{x}\right)$$

$$-\log H^+ = -\log K_a - \log\frac{1-x}{x}$$

$$pH = pK_a - \log\frac{1-x}{x}$$

$$pH - pK_a = \log\frac{x}{1-x}$$

64. The products A and B in the following reactions, respectively are

$$A \xleftarrow{Ag-NO_2} CH_3-CH_2-CH_2-Br \xrightarrow{AgCN} B$$

(1) $CH_3-CH_2-CH_2-NO_2$, $CH_3-CH_2-CH_2-CN$

(2) $CH_3-CH_2-CH_2-ONO$, $CH_3-CH_2-CH_2-CN$

(3) $CH_3-CH_2-CH_2-ONO$, $CH_3-CH_2-CH_2-NC$

(4) $CH_3-CH_2-CH_2-NO_2$, $CH_3-CH_2-CH_2-NC$

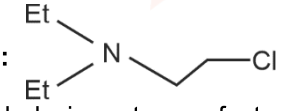
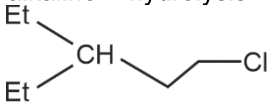
Answer (4)

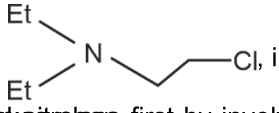
Sol. NO_2^- and CN^- are ambidentate nucleophiles

A : $CH_3-CH_2-CH_2-NO_2$

B : $CH_3-CH_2-CH_2-NC$

65. Given below are two statements:

Statement I:  will undergo alkaline hydrolysis at a faster rate than 

Statement II:  , intramolecular substitution takes place first by involving lone pair of electrons on nitrogen.

In the light of the above statements, choose the most appropriate answer from the options given below

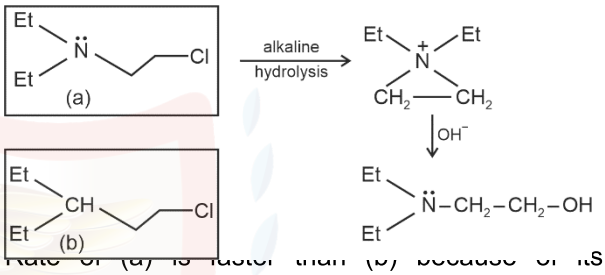
(1) Statement I is incorrect but Statement II is correct

(2) Statement I is correct but Statement II is incorrect

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Answer (4)

Sol. 

Rate of (a) is faster than (b) because of its intramolecular substitution.

66. What is the freezing point depression constant of a solvent, 50 g of which contain 1 g non volatile solute (molar mass 256 g mol^{-1}) and the decrease in freezing point is 0.40 K?

(1) 1.86 K kg mol^{-1} (2) 5.12 K kg mol^{-1}

(3) 4.43 K kg mol^{-1} (4) 3.72 L kg mol^{-1}

Answer (2)

Sol. $\Delta T_f = 0.40$ K

$$(0.40) = (K_f)\left(\frac{1000}{(256)(50)}\right)$$

$$\boxed{(K_F) = 5.12 \text{ K kg mole}^{-1}}$$

67. The metal ion whose electronic configuration is not affected by the nature of the ligand and which gives a violet colour in non luminous flame under hot condition in borax bead test is

(1) Ni^{2+} (2) Mn^{2+}

(3) Cr^{3+} (4) Ti^{3+}

Answer (1)

Sol. Ni^{2+} gives violet colour with borax bead test in non-luminous flame under hot conditions. Ni^{2+} has d^8 configuration which does not depend on nature of ligand present in octahedral field.

$Ni^{2+} \Rightarrow t_{2g}^6 eg^2$

68. Both acetaldehyde and acetone (individually) undergo which of the following reactions?

A. Iodoform Reaction

B. Cannizzaro Reaction

C. Aldol Condensation

D. Tollen's Test

E. Clemmensen Reduction

Choose the ***correct*** answer from the options given below:

(1) B, C and D only (2) A, B and D only

(3) A, C and E only (4) C and E only

Answer (3)

Sol.

	Name of Reaction	Acetaldehyde	Acetone
A.	Iodoform reaction	✓	✓
B.	Cannizaro reaction	×	×
C.	Aldol reaction	✓	✓
D.	Tollen's test	✓	×
E.	Clemmensen reduction	✓	✓

69. Given below are two statements :

Statement I : In the oxalic acid vs $KMnO_4$ (in the presence of dil H_2SO_4) titration the solution needs to be heated initially to 60°C, but no heating is required in Ferrous ammonium sulphate (FAS) vs $KMnO_4$ titration (in the presence of dill H_2SO_4)

Statement II : In oxalic acid vs $KMnO_4$ titration, the initial formation of $MnSO_4$ takes place at high temperature, which then acts as catalyst for further reaction. In the case of FAS vs $KMnO_4$, heating oxidizes Fe^{2+} into Fe^{3+} by oxygen of air and error may be introduced in the experiment.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Answer (4)

Sol. Heating is required in oxalic acid filtration due to High activation energy.

Heating is notrequired in FAS vs $KMnO_4$ titration because Fe^{2+} will get converted into Fe^{3+} by oxygen of air and error may be introduced in the experiment.

Both Statement-I and Statement-II are correct.

70. Consider the following elements In, Tl, Al, Pb, Sn and Ge.

The most stable oxidation states of elements with highest and lowest first ionisation enthalpies, respectively, are

(1) +2 and +3 (2) +4 and +3

(3) +4 and +1 (4) +1 and +4

Answer (2)

Sol. In, Tl, Al, Pb, Sn Ge

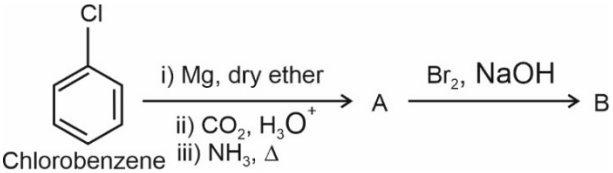
Lowest first I.E. = In $\Rightarrow$ most stable oxidation state $\Rightarrow$ +3

Highest first I.E. = Ge $\Rightarrow$ most stable oxidation state $\Rightarrow$ +4

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded off to the nearest integer.

71. Consider the following sequence of reactions:



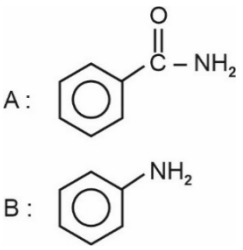
11.25 mg of chlorobenzene will produce ----- × 10^{-1} mg of product B. conversion.)

(Consider the reactions result in complete 16, 14 and 35.5 g mol^{-1} respectively]

[Given molar mass of C, H, O, N and Cl as 12, 1,

Answer (93)

Sol.



moles of chlorobenzene = $\frac{11.25\times10^{-3}}{112.5}$

= 10^{-4}

moles of product B = 10^{-4}

mass of product B = (10^{-4}) × 93

= 93 × 10^{-1} milligram

72. The formation enthalpies, ΔH_f° for $H_{(g)}$ and $O_{(g)}$ are 220.0 and 250.0 kJ mol^{-1}, respectively, at 298.15 K, and ΔH_f° for $H_2O_{(g)}$ is –242.0 kJ mol^{-1} at the same temperature. The average bound enthalpy of the O–H bond in water at 298.15 K is _____ kJ mol^{-1} (nearest integer).

Answer (466)

Sol. $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$

(–242) = 440 + 250 – 2x

2x = 440 + 250 + 242

x = 466

73. Quantitative analysis of an organic compound (X) shows following % composition.

C : 14.5% Cl : 64.46%

H : 1.8%

(Empirical formula mass of the compound (X) is _____ × 10^{-1}

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Cl : 35.5)

Answer (1655)

Sol. $C_{\left(\frac{14.5}{12}\right)}\ H_{\left(\frac{1.8}{1}\right)}\ Cl_{\left(\frac{64.46}{35.5}\right)}\ O_{\left(\frac{19.24}{16}\right)}$

$C_{1.21}\ H_{1.8}\ Cl_{1.81}\ O_{(1.2)}$

$\Rightarrow C_1\ H_{1.5}\ Cl_{1.5}\ O_1$

$\Rightarrow C_2H_3Cl_3O_2$

Empirical formula mass = $165.5\frac{gm}{mole}$

= 1655 × 10^{-1}

74. The molarity of a 70% (mass/mass) aqueous solution of a monobasic acid (X) is _____ × 10^{-1}

M(Nearest integer)

[Given : Density of aqueous solution of (X) is 1.25 g mL^{-1}

Molar mass of the acid is 70 g mol^{-1}]

Answer (125)

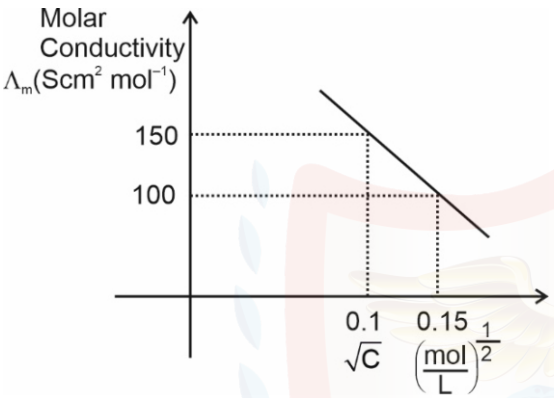
Sol. Moles of solute = $\frac{70}{70} = 1$

Volume of solution = $\frac{100}{1.25} = 80$ mL

$M = \frac{1}{80}\times1000 = 12.5$

M = 125 × 10^{-1}

75. Given below is the plot of the molar conductivity vs $\sqrt{\text{concentration}}$ for KCl in aqueous solution.



If, for the higher concentration of KCl solution, the resistance of the conductivity cell is 100 W, then the resistance of the same cell with the dilute solution is 'x' W

The value of x is _____ (Nearest integer)

Answer (150)

Sol.

$$100 = \rho\left(\frac{l}{A}\right)$$

For $\sqrt{C}$ = 0.15

l_m = 100

$$100 = \frac{k \times 1000}{0.15 \times 0.15}$$

$$k = \frac{15 \times 15}{10^5} = \frac{225}{105}$$

$$\rho = \frac{10^5}{225}$$

$$100 = \frac{10^5}{225}\left(\frac{l}{A}\right)$$

$$\frac{l}{A} = \frac{225}{1000}$$

For $\sqrt{C}$ = 0.1

l_m = 150

$$150 = \frac{k \times 1000}{10^{-2}}$$

$k = 15 \times 10^{-4}$

$$\rho = \frac{10^4}{15}$$

$$R = \rho\left(\frac{l}{A}\right)$$

$$R = \frac{10^4}{15} \times \frac{225}{1000}$$

$R \Rightarrow 150$ W

❑ ❑ ❑